# Why deep learning? Some technical and practical aspects

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AI? What is it? What for? What types?

Long-standing dream (back to the anciant Greeks)<sup>1</sup> of having intelligent artificial creatures.

C. Galiez (LJK-SVH)

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In particular, we will focus on (the distinction between):

- machine learning
- neural networks
- deep learning

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#### Controversies

#### In the media:

- + Al solve all problems: ecology, unemployment, etc.
- - Al is dangerous: "big data is watching you"
- Al is not fair: biases

Interesting article about biases in The Consversation.

In the scientific community:

- ullet + Al solves everything: you can predict anything if you have the data
- - Al does not explain anything: it's only black boxes

We will try today to get a fairer judgement on Al.

#### Be fair but critical



What is the right place of AI?

#### Manichean views

Think in background of two examples of AI applications (can be softwares, proof of concepts, etc.), one you would characterize as **good**, one as **bad**. Try to think in particular what would be the **societal impacts** if the examples you chose were generalized in the world. Let's check in 2h:)

# Today's outline

- Al? What for? What types?
- Machine learning
  - Learn from experience (data)
  - Underfitting (beware of biases)
  - Overfitting
- Neural networks
  - Properties
  - Optimizing the loss
  - Architecture matters
- Applications
- Timeline and future of Al
  - Biases
  - Ethics
  - Open problems

- Operational (automation of tasks)
  - Robotics (self-driving cars)
  - Indexing images, tagging
- Modeling
  - Extraction of information
  - Prediction

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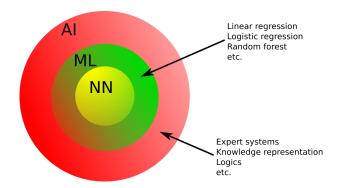
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#### Any idea?

- Operational (automation of tasks): can be done by a human. We trust the human more than the AI.
  - Robotics (self-driving cars)
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- Modeling: outcome can't be done by a human. We trust the AI more than human.
  - Extraction of information
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In both cases we are interested by the quality of the prediction, but may be also interested by bound guarantees or explanation of the "black-box".

### Al taxonomy



We will focus on the *data science* part of artificial intelligence : **machine learning**.

### Some machine learning methods

What machine learning tool you already know?

# Some machine learning methods

#### What machine learning tool you already know?

#### For classification tasks:

- Linear Discriminant Analysis (LDA)
- Logistic regression
- Support Vector Machine (SVM)
- Random forests
- Artificial neural networks

#### For regression tasks:

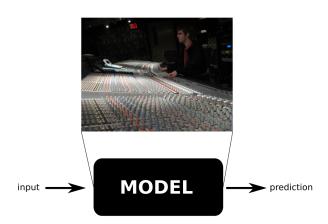
- Linear regression
- Regressive artificial neural networks

#### But also include parts of symbolic AI:

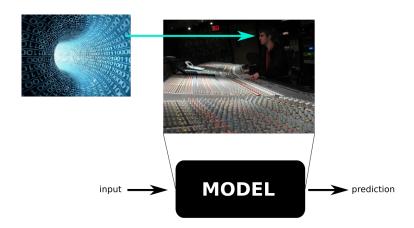
- Grammar inference
- Logic rule inference

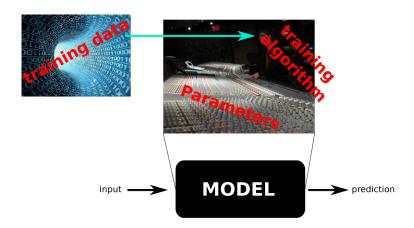






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<sup>&</sup>lt;sup>2</sup>[Brown et al. 20]

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 $2^{175.10^9}$ 

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If I test 10B possibility per second (10GHz), how many seconds are necessary to try out all possible combinations?

Number of possible combinations:

$$2^{175.10^9} = (2^{10})^{17.510^9}$$

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$$2^{175.10^9} = (2^{10})^{17.5 \cdot 10^9} \approx (10^3)^{17.5 \cdot 10^9} \approx 10^{52.000.000.000}$$

Computational time required:  $10^{52.000.000.000-10}$ 

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How it is possible to train such a model?

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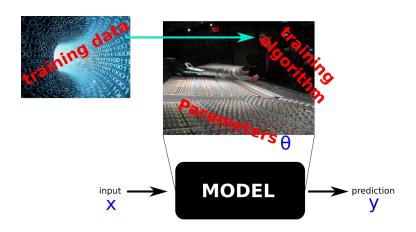
### Some order of magnitude: training data

• Recent GPT-3 NLP (Natural Language Processing) model: trained on a corpus of 300B tokens ( $\approx 1TB$ ).

Dataset	Quantity (tokens)	Weight in training mix	Epochs elapsed when training for 300B tokens
Common Crawl (filtered)	410 billion	60%	0.44
WebText2	19 billion	22%	2.9
Books1	12 billion	8%	1.9
Books2	55 billion	8%	0.43
Wikipedia	3 billion	3%	3.4

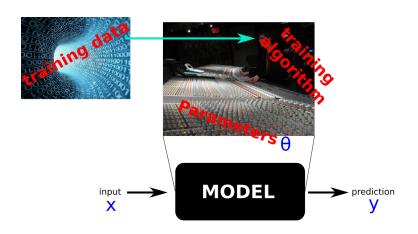
- Open Image Dataset V6 (18TB):
  - 9M images
  - 2M with labels from 600 classes

## Training ML models



What training data?

## Training ML models



What training data? Observations of (x, y):  $(x_1, y_1), ...(x_T, y_T)$ 



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You cannot measure it, because either:

You don't feel like it



(operational)



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#### **Problem**

You cannot measure it, because either:

You don't feel like it



(operational)

Nobody will do it



(modeling)

What to do?

What to do? Make a model :) !

We want to model the **fuel consumption** y (L/100km) with respect to the **car speed** x (in km/h).

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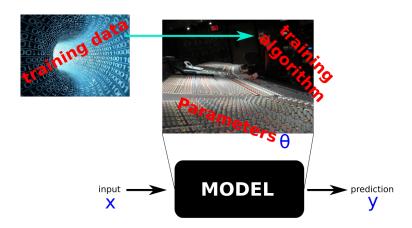
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What model could you use for the dependency between x and y?



Need to find the right  $\theta_0$  and  $\theta_1$ . We say we need to **fit** the model.

# Worked-out example: fitting



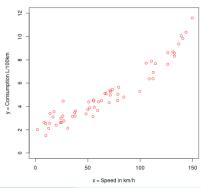
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# Worked-out example: the training data



Your neighbor, gives you her home-made measurements.

It consists in  $(x_i, y_i), i = 1, ..60$ 



Goal: find optimal  $\theta_0, \theta_1$  such that:  $y \approx \theta_0 + \theta_1.x$ 

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More formally,  $\theta_0, \theta_1$  such that

$$\mathcal{L}(\theta_0, \theta_1) = \frac{1}{60} \sum_{i=1}^{60} [y_i - (\theta_0 + \theta_1.x_i)]^2$$
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is minimal.

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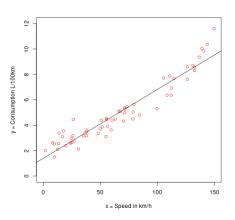
is minimal.

#### Definition

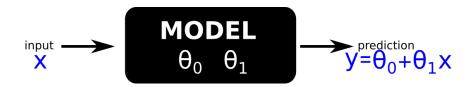
 $\mathcal{L}$  is called a **loss function** for the model<sup>a</sup>.

<sup>a</sup>This one in particular is called the *mean squared error* 

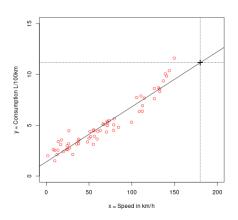
# Worked-out example: the fit

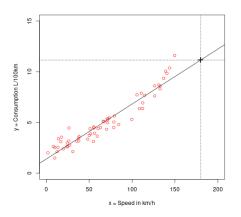


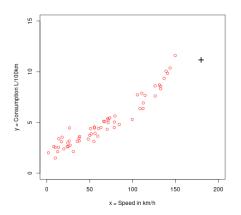
### Worked-out example: the prediction

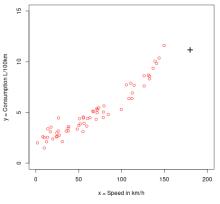


## Worked-out example: the prediction

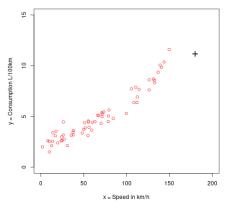








Any comment?



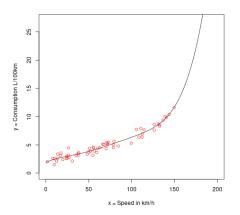
Any comment?
This phenomenon is known as **underfitting** 

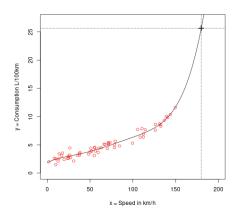


 $\theta_0, \theta_1..., \theta_5$  such that

$$\mathcal{L}(\theta_0, ..., \theta_5) = \frac{1}{60} \sum_{i=1}^{60} [y_i - (\sum_{j=0}^5 \theta_j . x_i^j)]^2$$
 (2)

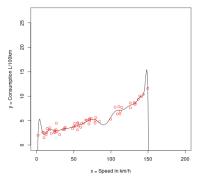
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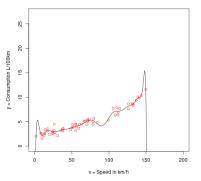


#### Informal definition

We will say that the polynomial model of degree 5 is more **expressive** than the linear regression.



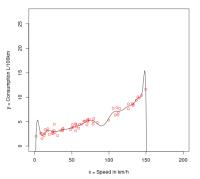
With 30 parameters:  $\theta_0, ... \theta_{29}$ 



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### Definition

The phenomenon is called **overfitting**.

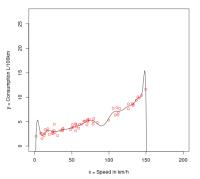


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### Definition

The phenomenon is called **overfitting**. Mainly happens because of **hyperparametrization**.

#### Parameters: the more the better?



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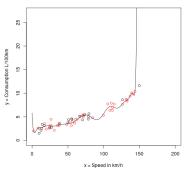






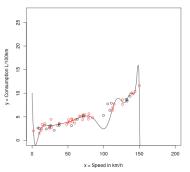


# Cross-validation example: 30 parameters



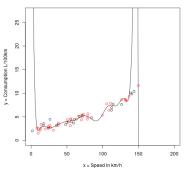
30 parameters, fold 1

### Cross-validation example: 30 parameters



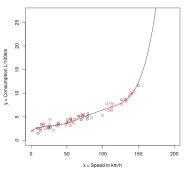
30 parameters, fold 2

### Cross-validation example: 30 parameters



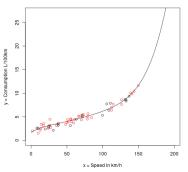
30 parameters, fold 3

### Cross-validation example: 6 parameters



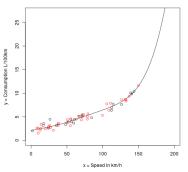
6 parameters, fold 1

### Cross-validation example: 6 parameters



6 parameters, fold 2

### Cross-validation example: 6 parameters



6 parameters, fold 3

- Lower error on training with \_\_\_parameters
- If the error is on the validation is much higher than on training set, it means that the model \_\_\_\_.
- More variance among fits with \_\_\_parameters<sup>3</sup>

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More specifically, one can decompose the error of the model as:

$$\mathbb{E}[||y - \hat{y}||^{2}] = \mathbb{E}[||y - \mathbb{E}[\hat{y}] + \mathbb{E}[\hat{y}] - \hat{y}||^{2}]$$

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$$= \text{bias}^{2} + \text{variance}$$
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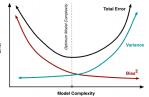
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This is known as the bias-variance trade-off:



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- We \_\_\_ a model by minimizing its \_\_\_ function on a \_\_\_ set.
- A model can have billion of \_\_\_ which make it \_\_\_ to try out all combinations.

- We **fit** a model by minimizing its \_\_\_ function on a \_\_\_ set.
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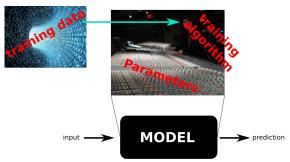
- We **fit** a model by minimizing its **loss** function on a \_\_\_ set.
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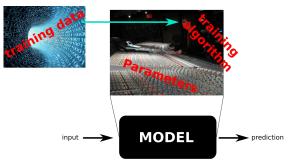
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- We fit a model by minimizing its loss function on a training set.
- A model can have billion of **parameters** which make it **impossible** to try out all combinations.

- We **fit** a model by minimizing its **loss** function on a **training** set.
- A model can have billion of parameters which make it impossible to try out all combinations.



- We fit a model by minimizing its loss function on a training set.
- A model can have billion of parameters which make it impossible to try out all combinations.



#### Remaining questions:

- What are the training algorithms?
- How is it possible to train with billions of parameters?

C. Galiez (LJK-SVH)

# Learning the parameters

# Learning algorithms

#### Of course...

Learning algorithm depends on the model to be fitted.

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Learning algorithm depends on the model to be fitted. But their job is to **minimize** a certain **loss**.

#### Examples:

- Small discrete models: enumeration and pruning
- Analytic solution for the minimum (e.g. linear regression)
- ullet Convex loss function o gradient descent methods
- EM algorithms
- etc.

### Loss with billions of parameters and datapoints

We would like  $\theta_0, \theta_1, \dots, \theta_p$  such that

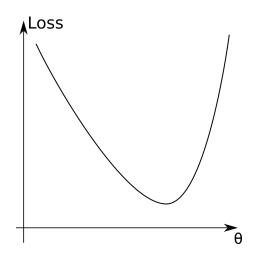
$$\mathcal{L}(\theta_0, ..., \theta_p) = \frac{1}{N} \sum_{i=1}^{N} [y_i - (\sum_{j=0}^{p} \theta_j . x_i^j)]^2$$
 (3)

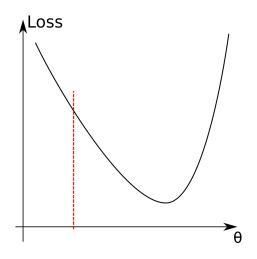
#### is minimal.

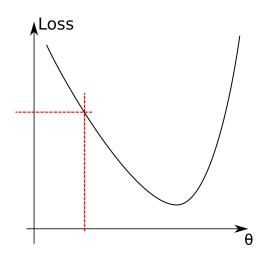
One would like to find the minimum of this loss.

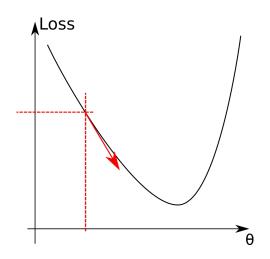
#### Issue: $p, n \sim 10^9$

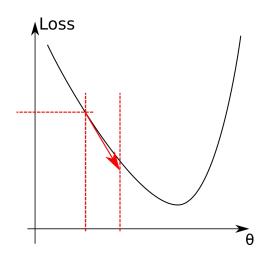
- Cannot test every parameter combination
- Every computation of the loss is costly.

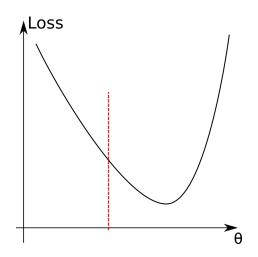


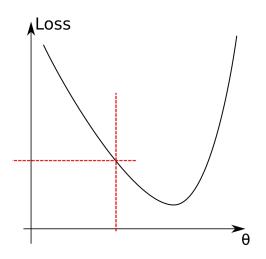


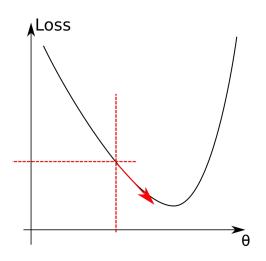


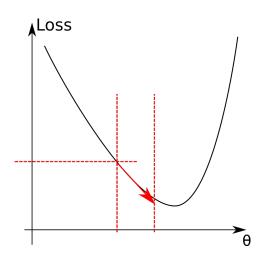


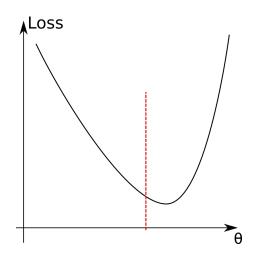


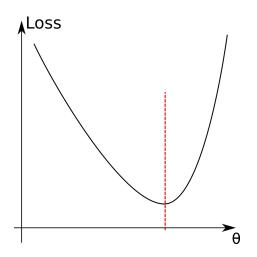


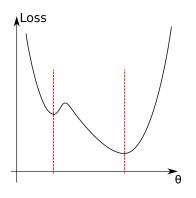




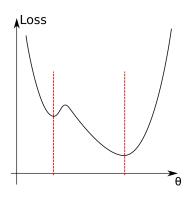








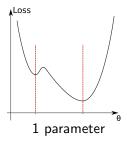
The gradient descent stops when it reaches a **critical point**:  $\nabla \mathcal{L}(\theta_n) = \vec{0}$ 



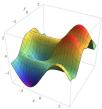
The gradient descent stops when it reaches a **critical point**:  $\nabla \mathcal{L}(\theta_n) = \vec{0}$ 

What are the possible types of critical points?

# Critical points in higher dimension

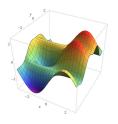


# Critical points in higher dimension



 $\geq 2$  parameters

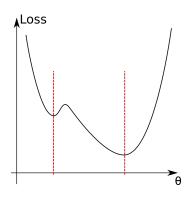
# Critical points in higher dimension



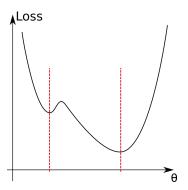
Туре	Hessian <sup>4</sup>
Local minimum	all eigenvalues $> 0$
Local maximum	all eigenvalues $< 0$
Saddle points	else



# Escaping easy local minima



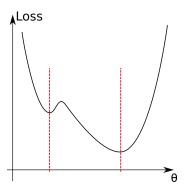
## Escaping easy local minima



Gradient descent works well in practice for "small" non convexities by adding an inertia term:

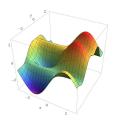
$$\begin{aligned} \operatorname{grad}_{n+1} &= \nabla \mathcal{L}(\theta_n) \\ \theta_{n+1} &= \theta_n - \eta_{n+1} \operatorname{grad}_{n+1} \end{aligned} \tag{4}$$

# Escaping easy local minima



Gradient descent works well in practice for "small" non convexities by adding an inertia term:

$$\begin{aligned} \operatorname{grad}_{n+1} &= \nabla \mathcal{L}(\theta_n) + \nu_{n+1}.\operatorname{grad}_n \\ \theta_{n+1} &= \theta_n - \eta_{n+1}\operatorname{grad}_{n+1} \end{aligned} \tag{4}$$

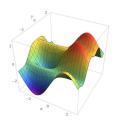


How is it possible to escape a critical point that is **not** a minimum?

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 $<sup>^5 \</sup>text{Would}$  need to invert a  $175.10^9$  dimensional matrix for GPT3...  $\approx 5.3.10^{33}$  operations :-/



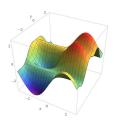
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Compute the Hessian

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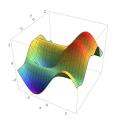
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How is it possible to escape a critical point that is **not** a minimum?

• Compute the Hessian (too big if a lot of parameters<sup>5</sup>)

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How is it possible to escape a critical point that is **not** a minimum?

- Compute the Hessian (too big if a lot of parameters<sup>5</sup>)
- Add "noise" to the gradient!

How to add some noise?

 $<sup>^5 \</sup>text{Would}$  need to invert a  $175.10^9$  dimensional matrix for GPT3...  $\approx 5.3.10^{33}$  operations :-/

# Stochastic gradient descent (SGD)

$$\mathcal{L}(\theta) = \sum_{i=1}^{N} l(\theta, x_i, y_i)$$

where  $\{(x_i, y_i)\}$  is the training set.

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where  $\{(x_i, y_i)\}$  is the training set.

If one computes instead the gradient on a **random subset**  $\mathcal B$  (coined a **batch**) of the training set, one has an unbiased<sup>6</sup> **noisy** estimate of the gradient:

$$\mathcal{L}_{\mathcal{B}}(\theta, X, Y) = \sum_{i \in \mathcal{B}} l(\theta, x_i, y_i)$$
 (5)

<sup>6</sup>By linearity of expectation:  $\mathbb{E}_{\mathcal{B}}[\mathcal{L}_{\mathcal{B}}(\theta, X, Y)] = \mathcal{L}(\theta, X, Y)$ 

# SGD properties

- SGD is guaranteed to converge to the minimum for a convex loss.
- SGD can escape easy saddle points
- ullet the computational cost is **much reduced**! See:  $N_{ ext{updates}} imes |\mathcal{B}|$
- with small batches can take advantage of modern hardware (GPUs)
- works very well in practice to find good local minima

# SGD properties

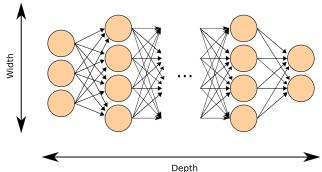
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It seems that we have a candidate to train a model with billions of parameters...

# **Neural Networks**

#### Neural networks

Dense feed-forward neural network:



Activation of neuron i in layer l:  $z_{i,l} = \sigma_j l(\sum_{k \in \mathcal{I}_{i,l}} w_{l,i,k} z_{k,l-1})$ 

Parameters<sup>7</sup>:  $w_{j,k}$ 's.

 $\sigma_l$ : activation functions.

<sup>7</sup>GPT-3 has 96 layers and 175B parameters

Inputs are vectors in  $\mathbb{R}^m$ , and output vectors in  $\mathbb{R}^n$ .

#### It can be:





out

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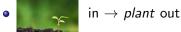


in o  $\mathit{plant}$  out

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ullet Draw me a plant in o out

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ullet in o plant out

ullet Draw me a plant in o out

$$ullet$$
 in  $o$  "plant" out

• ...

Images can be encoded as 1 px = 1 dimension of the vector, a signal as 1 time point = 1 dimension, etc.

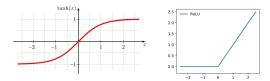
### Time to play

Let's see how an ANN looks like in practice: https://playground.tensorflow.org

#### Activation function

#### What activation function?

- Its gradient has to be simple to compute
- It should "make sense"



Note that the linear regression can be recovered with 1 single output neuron and f(x)=x as activation function.

### Neural networks as a "universal" model



#### **Theorem**

For any  $\epsilon > 0$ , and any Lebesgue-integrable function  $f : \mathbb{R}^m \to \mathbb{R}$ , there exists a ReLU neural network  $\eta$  such that:

$$\int |f(x) - \eta(x)| dx < \epsilon \tag{6}$$

Moreover, one can also restict the maximal width to m+4.

There are theoretical results ensuring that neural nets can approximate arbitrarily well any well-behaved function.

### Time to play

Let's see how ReLU and depth allow to model complex data: https://playground.tensorflow.org

# Training: SGD to the rescue!

#### Which training algorithm?

- + Automatic computation of the gradient of the loss using *automatic* differentiation.
- The loss is non-convex even with linear activation functions [Kawaguchi NIPS'16].

#### Nevertheless...

Gradient descent works very well in practice.

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#### Nevertheless...

Gradient descent works very well in practice.

We now have some theoretical results explaining (a bit) why it SGD works well for deep neural networks [Hardt et al. 16] (SGD and generalization error), [Chaudhari and Soatto ICLR'18] (SGD naturally regularizes).

# Training a neural network: the big picture

A (dense) NN is therefore simply a function

$$f_w(x) = \sigma_d(\sum_{k_d} w_{d,1,k_d} \sigma_{d-1}(\sum ... \sigma_1(\sum_{k_1} w_{1,i,k_1} x_{k_1}))).$$

How to fit the parameters  $w_{jkl}$ ?

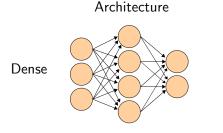
- Get a traning set  $(x_s, y_s)$ , can range from kB to TB of data<sup>8</sup>.
- Define a loss function, for instance  $\mathcal{L}(w) = \sum_{s} [y_s f_w(x_s)]^2$ .
- ullet Then compute  $^{9}$  the derivatives  $rac{\partial \mathcal{L}}{\partial w_{i,j}}$
- Use your favorite variant of SGD, and find a good minimum of the loss.

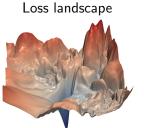
<sup>8</sup>the more the better

 $<sup>^{9}\</sup>mathrm{An}$  analytical formula can be obtain by a computer for well-chosen activation functions

## Architecture matters for training: resNet example

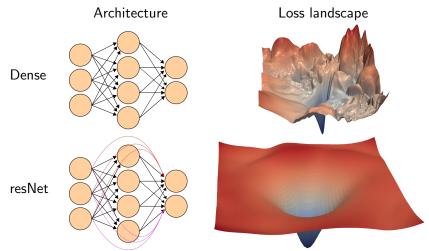
The loss function can change dramatically depending on the NN architecture.





#### Architecture matters for training: resNet example

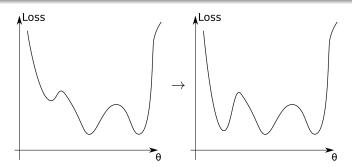
The loss function can change dramatically depending on the NN architecture.



#### Architecture matters for training: depth

#### (spin-glass model)

The bigger the network, the fewer bad local minima.



The loss of neural networks is
The loss landscape depends on the It therefore makes sense to use a
architecture for which local minima are of quality (like deep or specific
like ResNet).
So neural networks are universal models that can be trained
efficiently using But the deeper the ANN, the parameters are
involved so even with a huge load of data, it should, right?!

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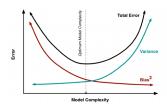
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So... **ReLU** neural networks are universal models that can be trained efficiently using **SGD**. But the deeper the ANN, the **more** parameters are involved... so even with a huge load of data, it should **overfit**, right?!

The reason why ANN tend not to overfit is not clear yet.

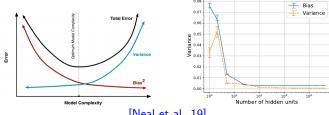
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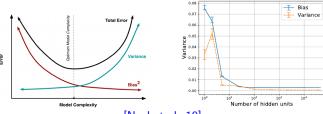
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[Neal et al. 19]

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[Neal et al. 19]

#### A big insight [Achille and Soatto JMLR'18]

One should rather measure the amount of information from the training data that is transferred to the weights during the fit: less and less amount is transferred the deeper you go.

#### Time to play

Let's see what neurons learn with respect to depth:

https://playground.tensorflow.org

No real "breakthrough" but rather a concordance of events:

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<sup>&</sup>lt;sup>10</sup>Actually it may be also why biological neural nets have been selected by evolution

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Better optimization algorithms (SGD and its variants)



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 Some luck<sup>10</sup>: ANN tend not to overfit (but it has been noticed afterwards)

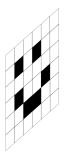
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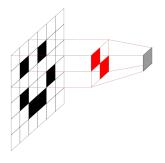
# Some architecture you need to know

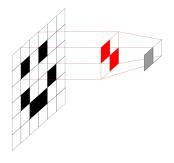
#### What architecture are allowed?

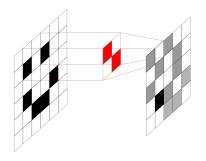
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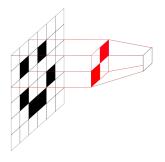
Virtually any, as soon as you can compute the gradient of the loss :)

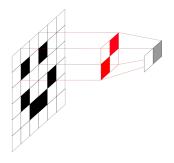


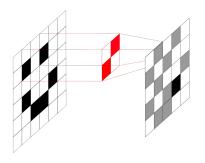


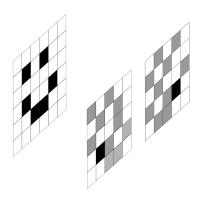


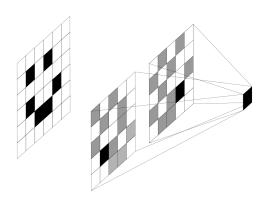












#### Real-world CNN



Source: InceptionV3

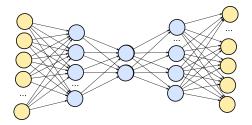
Applications: image recognition, segmentation [Minaee et al. 20], etc.

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## Time to play

Demo ResNet50.

### Compressing data with NN: Autoencoders



Try to accurately reconstruct the input (unsupervised).

Analogy with the "Chinese Whispers" 11. The mid-layer is called a latent and contains a compressed version of the input. Works when the data has an underlying structure.

<sup>11 &</sup>quot;Téléphone Arabe" in French

#### Compressing data: autoencoders

Example: predicting high altitude pollution from satellite images

#### **Problem**

- Cannot generate a lot of training data: only few ballon can be sent per year.
- hourly acquisition of 2000x2000px satellite images

What will happen if we learn the pollution from the raw sattelite images?

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What will happen if we learn the pollution from the raw sattelite images? **Overfitting** 

You can: **compress** the images of sattelite (trained on all unlabled data) and predict from the small **latent** layer.

Example: predicting high altitude pollution from satellite images

#### **Problem**

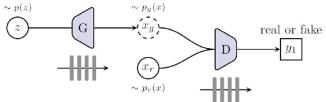
- Cannot generate a lot of training data: only few ballon can be sent per year.
- hourly acquisition of 2000x2000px satellite images

What will happen if we learn the pollution from the raw sattelite images? **Overfitting** 

You can: **compress** the images of sattelite (trained on all unlabled data) and predict from the small **latent** layer.

It is very common to have a lot of unlabled data and few labeled data.

## Generative Adversial Networks (GAN)



Equilibrium reached when  $p_q = p_r$ 

## Generative Adversial Networks (GAN)



Can be use as a generative process

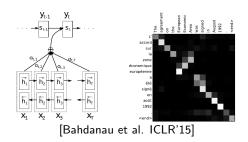
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## Generative Adversial Networks (GAN)



or as a classifier [Yi et al. 20]

### Text processing: attention mechanism

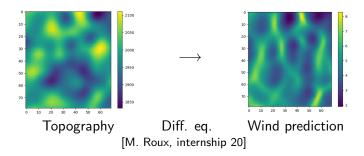


Improvement of the same idea: the Transformer architecture [Vaswani et al. 17] ,[Brown et al. 20]

# **Applications**

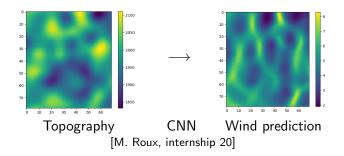
#### Surrogate models

Goal: use a neural network to approximate a costly model



#### Surrogate models

Goal: use a neural network to approximate a costly model



## Glaciers grow and melt



### Al and glacier evolution prediction

The causes influencing the evolution of glacier are complex:

- temperature
- solar radiation
- albedo of the glacier
- wind
- ...

**Yearly mass balance** can be estimated with physical models involving all these parameters.

### Physical parameters are hard to get

Measuring the physical parameters can be cumbersome.



...sometimes hard to evaluate (e.g. measuring properties of the ice).

Even with the fanciest physical model (that can also be wrong), the results can't be totally accurate.

#### Use unbiased and easy to measure proxy parameters

We can design a regression NN model so that we predict the mass balance:



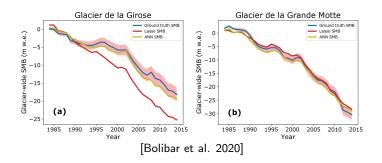
≈predicts



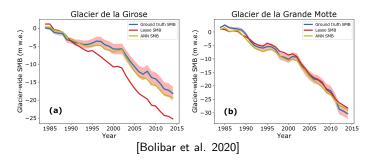
easy-measured features

mass balance

### Regression glacier model: better than linear



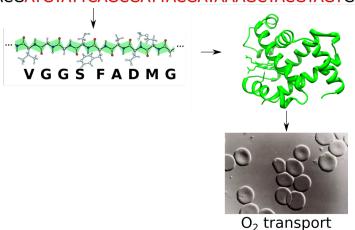
#### Regression glacier model: better than linear



Why not done before? Overfitting was hard to get rid of!

#### Protein sequence and structure

#### ACGATGTATTCAGCGATTACGATAAAGCTACGTAGTGGCA



O<sub>2</sub> cransport

### A recent big achievement: protein structure prediction

#### Goal: predict the structure from sequence

>1A3N:A|PDBID|CHAIN|SEQUENCE VLSPADKYNVKAMKOVGAHAGEYGAEALER MFLSPFTKYTPHFDLSHGSAOVKGHGKV ADALTNAVAHVDDMPNALSALSDLHAHKLRV DPVNFKLLSHCLLVTLAAHLPAEFTPAVHAS LDEPLASVETULTSKYB



>INEY-AIPBED (CHAIR I SEQUENCE WOODNINGE WOODNINGE WOODNINGE COAGESTAKE, AIPBED COAGESTAKE, AIPBED CHARACTER AIRBOTTER AIRBOTT



>IMCK.A. | POBED | CHARN | SEQUENCE MEMPOWERS | GEOTYOVYKARSKYLTGEVAL |
KKIELDYHTETGYPTAL BEI SLLKELANHONIV. |
KKIELDYHTETKLYLYPEPLADOLKKFADASAL |
TOFFLELIK STLFQLLGGLACKSHRVLARDL |
KROMELINTEGAL KLADPGLARAFOVYVETT |
KROMELINTEGAL KLADPGLARAFOVYVETT |
FAENVTHYKARSHLJGCKYYSTAVINSIGGI |
FAENVTHYKARSHLJGCKYYSTAVINSIGGI |
FAENVTHYKARSHVESFFKANAGOPSKVYPLD |
EOGSILSGMLHYDPNKRISAKAALAMPPPQD |
FORSILSGMLHYDPNKRISAKAALAMPPPQD |
FREYBYPHRKI |



#### **CASP** competition

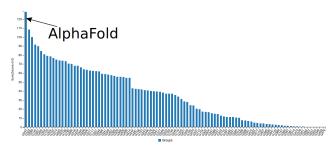
Blind competition. Simple principle:

- a sequence is given
- have to predict the structure.

Prior to 2018 it used to be (pseudo) physical models that where best performing.

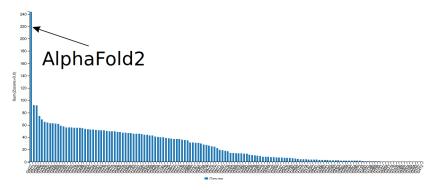
## CASP13 (2018)

Al wins the challenge for the first time.



## CASP14 (2020)

"The big leap forward"



Alphafold2: attention-based learning on protein sequence alignments [Casp14.]
Nature's article.

## Some fun and worrying examples: deepfake

link to video

#### Some useful and worrying examples: GPT3 examples

```
Poor English input: I eated the purple berries.

Good English output: I ate the purple berries.

Poor English input: Thank you for picking me as your designer. I'd appreciate it.

Good English output: Thank you for choosing me as your designer. I appreciate it.

Poor English input: The mentioned changes have done. or I did the alteration that you requested. or I changed things you wanted and did the modifications.

Good English output: The requested changes have been made. or I made the alteration that you requested. or I changed things you wanted and made the modifications.

Poor English input: I'd be more than happy to work with you in another project.
```

Poor English input: Please provide me with a short brief of the design you're looking for and that'd be nice if you could share some examples or project you did before. Good English output: Please provide me with a brief description of the design you're

took mgilsh output: Please provide me with a brief description of the design you're looking for and that would be nice if you could share some examples or projects you have done before.

Poor English input: The patient was died.
Good English output: The patient died.

Poor English input: We think that Leslie likes ourselves. Good English output: We think that Leslie likes us.



#### Some useful and worrying examples: GPT3 examples

Helsinki is the capital and largest city of Finland. It is in the region of Ususians, in southern Finland, on the shore of the Gulf of Finland. Helsinki has a population of , an urban population of , and a setropolitan population of year is shillow, making it the most populous municipality expected to the property of the property of

The Melsinki metropolitan area includes the urban core of Helsinki, Espoo, Vantaa, Kaumiainen, and surrounding commuter towns. It is the world's northermost metro area of over one million people, and the city in the area is the third largest metropolitan area in the Mordic countries after Stockholm and Copenhagen, and the City of Helsinki is the third largest after Stockholm and Golo. Helsinki is Finland's major political, educational, financial, cultural, and research center as well as one of northern Europe's major cities. Approximately 175, of foreign companies provided to the control of the control of

Q: what is the most populous municipality in Finland?

A: Helsinki

Q: how many people live there?

A: 1.4 million in the metropolitan area

 ${\tt Q}\colon$  what percent of the foreign companies that operate in Finland are in Helsinki?

A: 75%

That's impressive.

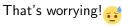


### Some useful and worrying examples: GPT3 examples

Title: United Methodists Agree to Historic Split Subtitle: Those who oppose gay marriage will form their own denomination Article: After two days of intense debate, the United Methodist Church has agreed to a historic split - one that is expected to end in the creation of a new denomination, one that will be "theologically and socially conservative," according to The Washington Post. The majority of delegates attending the church's annual General Conference in May voted to strengthen a ban on the ordination of LGBTO clergy and to write new rules that will "discipline" clergy who officiate at same-sex weddings. But those who opposed these measures have a new plan: They say they will form a separate denomination by 2020, calling their church the Christian Methodist denomination. The Post notes that the denomination, which claims 12.5 million members, was in the early 20th century the "largest Protestant denomination in the U.S.," but that it has been shrinking in recent decades. The new split will be the second in the church's history. The first occurred in 1968, when roughly 10 percent of the denomination left to form the Evangelical United Brethren Church. The Post notes that the proposed split "comes at a critical time for the church, which has been losing members for years," which has been "pushed toward the brink of a schism over the role of LGBTQ people in the church." Gay marriage is not the only issue that has divided the church. In 2016, the denomination was split over ordination of transgender clergy, with the North Pacific regional conference voting to ban them from serving as

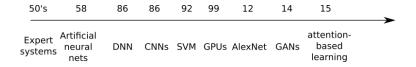
clergy, and the South Pacific regional conference voting to allow them.

Figure 3.14: The GPT-3 generated news article that humans had the greatest difficulty distinguishing from a human written article securacy: 12%.



## Past and future

#### **Timeline**



#### **Biases**

- Correlation ≠ causality
  - Pneumonia and asthma example [Crawford and Calo 16]
- Minorities in training data
  - less accurate for minorities [demo: beard-face on ResNet50]
- Models can increase biases
  - Underfitting and overfitting
  - preceptron for hiring

#### Al and CO<sub>2</sub>

Al can consumes a lot of electrical energy, having a strong environmental impact. Here are some figures showing the equivalent CO2 emission for creating some famous Al models for natural language processing:

Model	Hardware	Power (W)	Hours	kWh·PUE	$CO_2e$	Cloud compute cost
Transformer <sub>base</sub>	P100x8	1415.78	12	27	26	\$41-\$140
Transformer <sub>biq</sub>	P100x8	1515.43	84	201	192	\$289-\$981
ELMo	P100x3	517.66	336	275	262	\$433-\$1472
$BERT_{base}$	V100x64	12,041.51	79	1507	1438	\$3751-\$12,571
$BERT_{base}$	TPUv2x16	_	96	_	_	\$2074-\$6912
NAS	P100x8	1515.43	274,120	656,347	626,155	\$942,973-\$3,201,722
NAS	TPUv2x1	_	32,623	_	_	\$44,055-\$146,848
GPT-2	TPUv3x32	_	168	_	_	\$12,902-\$43,008

Table 3: Estimated cost of training a model in terms of CO<sub>2</sub> emissions (lbs) and cloud compute cost (USD). Power and carbon footprint are omitted for TPUs due to lack of public information on power draw for this hardware.

[Strubell et al. 19]

## Conclusion

### Open problems

- (Fully) understand generalization ability of deep NN
- How to improve collaboration/reuse of models in AI?
  - Model distillation (big model → small model)
  - Transfer learning (application  $A \rightarrow application B$ )
- How to reduce learning hassle?
  - Unsupervised learning
  - Few-shot learning
- How to have guarantees?
  - Explainable AI
- How to reduce/remove biases (disentangle correlation and causality)
- How to regulate the creations/usages of AI<sup>12</sup>?

<sup>&</sup>lt;sup>12</sup>Cannot rely on companies for this. See here.

### To sum up: what can I do with DL?

As soon as you have data, either labeled or unlabled, you can learn a model (in particular a DNN).

If there is some  $information^{13}$  in your training set, there is a good chance that the model will learn it.

You can use this model to predict on further data, to take decision, to estimate values, etc.

Note that now, most of the basic tasks (segmentation, image classification, etc.) can be achieved using pre-trained models. You can adapt your model to your specific dataset (few-shot learning).

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C. Galiez (LJK-SVH) Why deep learning? February 24, 2021

 $<sup>^{13}</sup>$ for instance between the data and the labels, or a structure underlying the data

#### The good, the bad, and the ugly

What applications would you consider as beneficial or detrimental for the society?

### The good, the bad, and the ugly

What applications would you consider as beneficial or detrimental for the society?

Backfire effect/Jevons paradox



- Less humanistic considerations
- Biases
- "Unresponsibilizing"
- Pushes society toward technology
- Automation of (boring) tasks
- Prevents from human mistakes
- Allow extract (unseen) information from data

## Discussion and questions?

#### Regularization

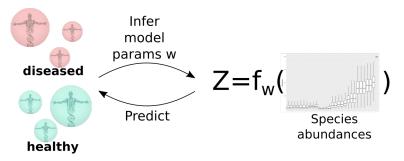
Regularization is an important technique that aims at excluding unrealistic parameter combinations.

- Ridge regularization: avoids big values of parameters
- Lasso regularization: favor nullity of parameters (parcimonious model)
- Bayesian modeling: model a priori knowledge on each parameter

# Application: example in health

## MWAS: metagenome-wide association studies

We can build models to predict diseases from microbial abundances, a process known as MWAS:



## MWAS as a classification problem

#### Let:

- ullet  $ec{X}$  be an M-dimensional random vector of abundance of species,
- and Z binary (0/1) random variable describing the disease state of a human.

Define a predictor  $f: \mathbb{R}^M_+ \to [0,1]$  such that it minimizes a *loss* on a training set  $(\vec{x}_1,z_1),...,(\vec{x}_N,z_N)$ :

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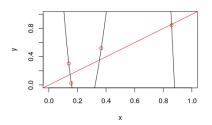
$$\min_{f} - \sum_{i=1}^{N} z_{i} \cdot \log f(\vec{x}_{i}) + (1 - z_{i}) \cdot \log(1 - f(\vec{x}_{i}))$$

## Regularization

### Ridge regularization example

Let's come back to the model  $Y = \sum_{i=0}^{3} \beta_i x^i + \epsilon$ .

The maximum likelihood with 4 points will give a  $\vec{\beta}$  fitting perfectly the points:



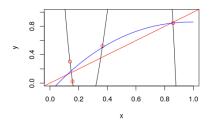
#### Maximum likelihood coefficients:

$$\beta_0$$
  $\beta_1$   $\beta_2$   $\beta_3$  5.169 -54.388 155.755 -114.487

### Ridge regularization example

Let's come back to the model  $Y = \sum_{i=0}^{3} \beta_i x^i + \epsilon$ .

With a prior  $\mathcal{N}(0, \eta^2)$  the maximum a posteriori of the vector  $\vec{\beta}$  corresponds to (blue curve):



#### Maximum a posteriori coefficients

$$\beta_0$$
  $\beta_1$   $\beta_2$   $\beta_3$  -0.1279 2.2561 -1.5779 0.3180

#### Quizz

#### Overfitting depends on:

- Size of the training set
- Complexity of the problem
- The parametrization of the model
- The type of the model